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The Harmonic Radiation Notes

A signed series, the catastrophe it does not solve,
and the piece that belongs in the program

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Abstract. Notes accumulated over several years try to replace Planck's energy elements with a harmonic series, so that blackbody radiation would be a classical continuum and a later single-system Hamiltonian could be built outside Hilbert space. The series goes negative wherever the sine does. An absolute value would hide the sign and destroy the interference the notes also want. The convergence of $\sum 1/n^2$ is not the ultraviolet problem. The latest formula reimports the Planck factor it was meant to avoid, and uses the letter k for two constants. Read against the 1949 program, the notes are an ancestor of the Fourier remainder in the Rank-1 analog, not a path that reaches a decay time.

1. Aim

If an individual emission is going to be a fact about a field, the radiation has to be written as a continuum, not as a vector that is updated by a second postulate. Planck's resonators were harmonic oscillators. The notes start there and try to keep every frequency, with a trigonometric series in place of $\varepsilon = h\nu$. The later hope is a Hamiltonian for one system in time, built from that radiation, and only then an instant of decay. That aim matches the program's refusal of Hilbert space as ontology. The formulae do not carry the aim.

2. The expressions

The early toy calls a conditionally convergent signed sum an energy:

$$E(\nu) = A \sum_{n=1}^{\infty} (1/n) \sin(n\nu).$$

The next version forces absolute convergence:

$$u(\nu) = A \sum_{n=1}^{\infty} (1/n^2) \sin(n\nu).$$

The latest version multiplies by the Planck factor and a damping exponential. In the notes the Boltzmann constant and the damping coefficient were both called k. They are not the same constant. Written apart:

$$u(\nu, t) = A(t) \sum_{n=1}^{\infty} (1/n^2) \cdot (h\nu)/(\exp(h\nu/k_B T) - 1) \cdot \sin(n\nu) \cdot \exp(-\kappa\nu).$$

$A(t)$ is an overall scale. n is a harmonic index, not Planck's integer of energy elements. κ is an absorption coefficient put in by hand.

3. The sign

For positive A, h, ν, T, and κ, every factor except the sine is positive. Term n has the sign of $\sin(n\nu)$. A direct sum of four thousand terms of $\sin(n\nu)/n^2$ is about $+1.014$ near $\nu = 1.2$ and about -1.015 near $\nu = 5.24$. It vanishes at $\nu = \pi$. Spectral radiance is an energy density. It cannot be negative. Neither can a photon energy.

An absolute value, or a square root of a square, would make a plot sit above zero. It would also remove the destructive half of any superposition. A classical field that interferes is supposed to change sign. The energy is

not that field. It is quadratic in the field and in its rate, and that quadratic is where non-negativity belongs. The notes promoted a displacement to an energy. Clipping the displacement is not the repair. Demoting it back to an amplitude is.

The double-slit remark in the notes is the part that survives this demotion. Constructive and destructive interference are what a signed amplitude is for. They are ordinary properties of a classical wave. They do not require this series, and they do not yield the instant at which a radioactive atom writes a mark.

4. Two infinities

The index n builds overtones of one fundamental. $\sum 1/n^2$ converges. That is a fact about one waveform, the same fact as the convergence of the series for a plucked string. It says nothing about how many independent modes a cavity supports between ν and $\nu + d\nu$.

The catastrophe, under that later name, is an integral. Equipartition assigns energy of order kT to each quadratic term. The modes contribute a factor ν^2. The energy per unit frequency grows without a bound as ν grows, and the integral diverges. Killing the overtones of one oscillator does not stop the next oscillator, at a higher frequency, from taking its share. The series solves a divergence the cavity does not have.

The factor $h\nu/(\exp(h\nu/k_B T) - 1)$ does cut the high frequencies off. It is Planck's mean energy, inserted, not derived. The damping $\exp(-\kappa\nu)$ is a second cutoff, also inserted. A model that contains the Planck factor has not replaced the energy element. It has decorated it with a sine.

5. Can the radiation be fully continuous

Yes. A Fourier integral $\int A(\nu)\sin(2\pi\nu t + \theta_\nu)d\nu$ is a classical field. No harmonic index is required, and no h is required, to write it down. Equilibrium in a cavity is a separate question. If the energy is a sum of quadratic terms and each term has mean $kT/2$, the spectrum is Rayleigh–Jeans and the integral diverges. That consequence is what forced Planck's element. Refusing musical discreteness does not refuse it. Accepting both a continuous waveform and a discrete set of overtones is ordinary Fourier analysis. It is not the joint possession of Planck's quantum and a continuum.

A new constant h' used as $\varepsilon = h'\nu$ inside a Boltzmann factor is the same element under a new letter. A constant already sitting in a field equation, a stiffness or a length, would be a different proposal, and only if it changed the equilibrium measure without chopping energy into steps. The notes do not derive any such measure. They sum a series.

6. Relation to the program

Shields 2026a already uses a finite Fourier background with definite phases as the unread remainder of one metastable well. The quantity that crosses a threshold is a positive functional of that field, not the signed sum. T^* is the first crossing. Relative phase in the same sort of field is enough to talk about interference. The notes are earlier than that construction and do not improve it.

They are not a discarded path that would have reached the radioactive instant if the sign had been repaired. As an amplitude, the series has no metastable structure and no threshold. As an energy, it is negative on a set of positive measure and, in the latest version, assumes the spectral law it was written to explain. A classical Hamiltonian outside Hilbert space remains a legitimate object for the program. It has to be bounded below, it has to be a functional of a continuous field, and the individual event has to be a crossing in one solution rather than an eigenvalue. Inserting the Planck factor into $\sum \sin(n\nu)/n^2$ does not produce that object. The derivation of that Hamiltonian is the History chapter, not a second copy of these notes.

7. References

The notes are the author's. The primary sources they are measured against are these.

1. M. Planck, “Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum,” *Verhandlungen der Deutschen Physikalischen Gesellschaft* 2, 237–245 (1900); *Annalen der Physik* 4, 553–563 (1901). The factor the latest note reimports.
2. W. Wien, “Über die Energievertheilung im Emissionsspectrum eines schwarzen Körpers,” *Annalen der Physik* 58, 662–669 (1896). The exponential tail the damping imitates.
3. Lord Rayleigh, *Philosophical Magazine* 49, 539–540 (1900); J. H. Jeans, *Philosophical Magazine* 10, 91–98 (1905). The equipartition spectrum the series does not cancel.
4. P. Ehrenfest, *Annalen der Physik* 36, 91–118 (1911). The name ultraviolet catastrophe, for the divergent integral, not for $\sum 1/n^2$.
5. W. Shields, “Individual Completion of Metastable Decay,” Zenodo (2026a). The Fourier remainder and T^* the notes do not reach.

Not a Zenodo deposit. The numerical range quoted for the sine series is from a partial sum of 4000 terms. Does not replace Shields 2026a or 2026b.