WORKING PAPER · NOT ON ZENODO

The Algorithm and the Individual

Schrödinger evolution, the measurement update,
and what the Bell record does not display

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Companion exhibition to Shields 2026a and Shields 2026b.
Those two records remain the program papers on Zenodo.
This paper does not replace them and is not deposited there.

Abstract. The two program papers convert Einstein's 1949 demand into a first-event time $T^*(S)$ and locate that demand in a field completion rather than in postwar unification. They do not exhibit, symbol by symbol, the algorithm they refuse to treat as a complete description of one individual. This paper does that exhibition. It writes the Schrödinger equation, the Born rule, and the measurement update, and it states what each symbol equals in a worked spin example. It then shows the empirical reason for the update: an ideal repeatable measurement is never a click that is half yes and half no, and a second measurement of the same observable agrees with the first. The update that secures this agreement is not a solution of the Schrödinger equation. That incompatibility is the measurement problem. Einstein's stance is then stated as he stated it: the algorithm is not internally contradictory; it is incomplete as a description of the individual, if one also refuses action at a distance. Bell's theorem and the later experiments are reviewed as constraints on click frequencies, not as photographs of a superposition. The correlations are real. The ontological reading that a linear ψ was on display is not. No new experimental gap inside those tables was found. The gaps that fit the suspicion that we over-read the formalism are explanatory: two laws are used where one is advertised, ensemble frequencies are spoken of as properties of one unmeasured run, and a reconstructed vector is reported as if it had been seen. A local unread remainder of the sort already defined in 2026a does not reproduce the Bell tables. That limitation is recorded, not argued away.

1 How this paper sits beside the other two

Shields 2026a, *Individual Completion of Metastable Decay*, states axioms for an individual description and defines

$$T^*(S) = \inf\{t > t_0 : E[\varphi_S](t) \geq E_c\}$$

for a complete state $S = S_{\text{coarse}} \oplus S_\perp$. Shields 2026b, *Completion, Not Unification*, separates that demand from quantization-as-unification and marks contact with already catalogued quantum statistics as a graduation constraint, not as an entrance examination written in Bell's variables.

This paper is the exhibition those two presuppose and do not give. It is a map of the predictive algorithm: what is calculated, what is updated, what is observed, and what is only named. It does not derive a field equation. It does not claim that a Rank-1 analog has passed a Bell test. It does not withdraw step 6 of 2026a, which remains open.

Three rules of reading:

1. A symbol is not used until it has been assigned.
2. A probability is a number computed before a click, or a frequency counted after many clicks. It is not itself a click.
3. "The experiment shows entanglement" is unpacked into counts, settings, and the assumptions of a theorem. The noun is not left to do the work.

2 The question, in one paragraph

Take the predictive algorithm at face value. Between measurements the state vector changes by the Schrödinger equation. At a measurement it is replaced by an eigenvector, with probabilities given by Born's rule. If that replacement is omitted, the algorithm describes something we do not observe: a second, immediate measurement of the same quantity that still comes out either way. If the replacement is kept, the algorithm no longer has one law. The question of this paper is what that split commits us to, once Einstein's completeness demand and the Bell record are both taken seriously, and whether any part of the record is an artifact of assuming that the linear ψ was the thing on the bench.

3 Objects, before dynamics

3.1 The space a state lives in

A **Hilbert space** $\mathcal{H}$ is a complex vector space with an inner product, complete in the norm that inner product defines. For every example in this paper except the short remarks on a particle in space, $\mathcal{H}$ is $\mathbb{C}^2$ or $\mathbb{C}^2 \times \mathbb{C}^2$. Those are finite. Nothing about completeness of infinite-dimensional spaces is required to see the measurement problem.

A **state vector** is a nonzero vector $|\psi\rangle \in \mathcal{H}$, taken up to a global phase: $|\psi\rangle$ and $e^{i\alpha}|\psi\rangle$ are the same state when α is a real constant. One normalizes

$$\langle \psi \,|\, \psi \rangle = 1.$$

A **basis** is a set of orthonormal vectors that span $\mathcal{H}$. For a spin measurement along z the basis used here is

$$|\mathrm{up}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\mathrm{down}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

These are eigenvectors of the operator written in the next section. They are not, by themselves, little arrows in the laboratory. They are the labels of the two outcomes that apparatus is built to distinguish.

3.2 Bra-ket notation, and what each mark is

Dirac's notation (Dirac 1939) is a way of writing vectors, dual vectors, inner products, and outer products so that the type of each object stays visible.

Mark	What it is, and what it equals here
$\|\psi\rangle$	A column vector. In the z-basis, $\|\psi\rangle = c_{\mathrm{up}}\|\mathrm{up}\rangle + c_{\mathrm{down}}\|\mathrm{down}\rangle = \begin{pmatrix} c_{\mathrm{up}} \\ c_{\mathrm{down}} \end{pmatrix}$, with $c_{\mathrm{up}}, c_{\mathrm{down}} \in \mathbb{C}$.
$\langle\varphi\|$	The conjugate row. If $\|\varphi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$, then $\langle\varphi\| = \begin{pmatrix} a^* & b^* \end{pmatrix}$.
$\langle \varphi \,\|\, \psi \rangle$	The inner product, a single complex number: $a^* c_{\mathrm{up}} + b^* c_{\mathrm{down}}$. If both vectors are normalized and $\langle \varphi \,\|\, \psi \rangle = 0$, they are orthogonal: the apparatus that accepts one rejects the other.
$\|c\|^2$	For a complex number, $c^* c$. A real number ≥ 0. This is the quantity Born's rule uses. It is not an outcome.
$\|\psi\rangle\langle\varphi\|$	An operator. Acting on $\|\chi\rangle$ it returns $\|\psi\rangle$ times the number $\langle \varphi \,\|\, \chi \rangle$.
I	The identity. In this basis $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
$A^\dagger$	The adjoint: conjugate transpose. $\sigma_z^\dagger = \sigma_z$. Observables are self-adjoint.
$\langle A \rangle_\psi$	The expectation $\langle\psi\|A\|\psi\rangle$. A number. Not the outcome of one run.

Global phase does not change any probability below. Relative phase does. That distinction is the entire content of interference.

3.3 Operators that stand for quantities

An **observable** is a self-adjoint operator $A = A^\dagger$. In a finite dimension it has real eigenvalues and an orthonormal basis of eigenvectors:

$$A|a_n\rangle = a_n|a_n\rangle, \quad a_n \in \mathbb{R}.$$

The number a_n is the value the algorithm is allowed to print when it says the outcome was n. Nothing else is printed. In particular the algorithm never prints "0.5" as the outcome of a two-outcome observable whose eigenvalues are ± 1.

For a spin-$\frac{1}{2}$ degree of freedom the three Pauli operators, in the z-basis, are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Spin angular momentum is $\vec{S} = (\tfrac{\hbar}{2})\vec{\sigma}$. The eigenvalues of S_z are $+\tfrac{\hbar}{2}$ and $-\tfrac{\hbar}{2}$. It is harmless, and usual, to compute with σ_z and restore $\tfrac{\hbar}{2}$ only when a joule or a magnet is required.

The eigenvectors of σ_x are

$$|{+}\mathrm{x}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |{-}\mathrm{x}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Check: $\sigma_x|{+}\mathrm{x}\rangle = |{+}\mathrm{x}\rangle$. So "$+$ along x" is the vector whose z-components are equal, not a vector that is already an eigenvector of σ_z.

4 The Schrödinger equation

4.1 The equation

For a state vector that depends on time and on no other variable,

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle.$$

If the Hamiltonian does not depend on time, the solution is

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \quad U(t) = \exp\!\left(-iH\frac{t}{\hbar}\right).$$

$U(t)$ is unitary: $U^\dagger U = I$. Norms and inner products of evolved vectors are preserved. Linear combinations evolve term by term. That linearity is not an extra mood. It is what the equation says.

4.2 Each symbol, and what it equals in the spin example

Take a magnetic moment in a constant field along z, with Hamiltonian

$$H = \frac{\hbar\omega}{2}\sigma_z = \begin{pmatrix} \hbar\frac{\omega}{2} & 0 \\ 0 & -\hbar\frac{\omega}{2} \end{pmatrix}.$$

The symbols in $i\hbar|\dot{\psi}\rangle = H|\psi\rangle$ then equal the following.

Symbol	Assignment in this example
i	The imaginary unit. $i^2 = -1$. Not a variable.

$\hbar$	$\frac{h}{2\pi}$, with $h = 6.62607015 \times 10^{-34}$ J s exact in the SI. Numerically $\hbar = 1.054571817 \times 10^{-34}$ J s.
t	Time, in seconds, the parameter along which the state is differentiated. Not an operator.
$\lvert\psi(t)\rangle$	$c_{\text{up}}(t)\lvert\text{up}\rangle + c_{\text{down}}(t)\lvert\text{down}\rangle$. Two complex functions of t, with $\lvert c_{\text{up}}\rvert^2 + \lvert c_{\text{down}}\rvert^2 = 1$.
d/dt	The ordinary derivative of those two functions. There is no spatial derivative in this example.
H	The matrix displayed above. Its two diagonal entries are the two allowed energies.
ω	A real angular frequency, in s^{-1}, fixed by the field and the moment. It is part of the apparatus, not an outcome.
σ_z	$\left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$. Eigenvalue $+1$ on $\lvert\text{up}\rangle$, eigenvalue -1 on $\lvert\text{down}\rangle$.

Componentwise the equation is two lines, and only two:

$$i\hbar \dot{c}_{\text{up}} = \left(\hbar\frac{\omega}{2}\right) c_{\text{up}}, \quad i\hbar \dot{c}_{\text{down}} = -\left(\hbar\frac{\omega}{2}\right) c_{\text{down}}.$$

Cancel $\hbar$ (it is not zero) and integrate:

$$c_{\text{up}}(t) = c_{\text{up}}(0)\exp\!\left(-i\omega\frac{t}{2}\right), \quad c_{\text{down}}(t) = c_{\text{down}}(0)\exp\!\left(+i\omega\frac{t}{2}\right).$$

That is the whole solution. No probability has entered. No outcome has been selected.

4.3 How to use it

The use is always the same sequence.

1. Name $\mathcal{H}$ and a basis tied to the outcomes the apparatus can print.
2. Write H as a matrix or a differential operator in that representation.
3. Write the initial vector, including relative phases.
4. Solve $i\hbar\lvert\dot{\psi}\rangle = H\lvert\psi\rangle$, or apply $U(t)$.
5. Stop. The solution is a vector at each t. It is not yet a list of clicks.

Example A. Precession, and what does not change. Let $\lvert\psi(0)\rangle = \lvert{+}\text{x}\rangle$, so $c_{\text{up}(0)} = c_{\text{down}(0)} = \frac{1}{\sqrt{2}}$. Then

$$\lvert\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega\frac{t}{2}} \\ e^{+i\omega\frac{t}{2}} \end{pmatrix}.$$

The two absolute squares are $\frac{1}{2}$ at every t. The relative phase ωt moves. A later measurement of σ_x feels that phase. A measurement of σ_z does not: both populations stay $\frac{1}{2}$. The equation has moved a phase. It has not chosen an outcome.

Example B. Two energies, a particle in a well. For a particle on an interval of length L with $V = 0$ inside and infinite walls, the spatial operator is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2},$$

with $\psi(0,t) = \psi(L,t) = 0$. Here m is the mass, x the coordinate, and d^2/dx^2 the second derivative. Stationary states are

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\!\Big(n\pi\frac{x}{L}\Big), \quad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}, \quad n = 1, 2, 3, \ldots$$

A superposition $|\psi(0)\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}}$ evolves as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\Big(e^{-iE_1\frac{t}{\hbar}}|1\rangle + e^{-iE_2\frac{t}{\hbar}}|2\rangle\Big).$$

The probability density $|\psi(x,t)|^2$ oscillates at the Bohr frequency $\frac{E_2-E_1}{\hbar}$. Each term still evolves. Neither term is deleted. This is the equation doing the only thing it can do: carrying every component forward.

Example C. Hydrogen, as a use, not a derivation. For an electron with

$$H = \frac{p^2}{2m_e} - \texttt{frac}\{e^2\}\{4\pi\varepsilon_0 r\},$$

the bound eigenvalues are $E_n = -13.6\frac{\text{eV}}{n^2}$. The symbols: $p = -i\hbar\nabla$ in the position representation, m_e the electron mass, e the elementary charge, ε_0 the vacuum permittivity, r the distance from the proton treated as fixed. This paper does not re-derive that spectrum. It is the standard illustration that the same equation, with a different H, is the one used to predict spectral lines. The lines are frequencies of an ensemble of emissions. The equation does not by itself say which atom emits at which instant. That is the gap 2026a is built to name, not a defect discovered in the algebra of E_n.

Example D. A joint system. Two spins have $\mathcal{H} = \mathbb{C}^2 \times \mathbb{C}^2$, with product basis $|\text{up up}\rangle, |\text{up down}\rangle, |\text{down up}\rangle, |\text{down down}\rangle$. The singlet

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\text{up down}\rangle - |\text{down up}\rangle)$$

is one vector in the joint space, not a pair of vectors. If the joint Hamiltonian is zero, Schrödinger's equation says this vector does not change. It does not say which side will read up.

5 The Born rule

5.1 The rule

For a nondegenerate observable with eigenvectors $|a_n\rangle$ and a normalized state,

$$P(a_n) = |\langle a_n \,|\, \psi\rangle|^2.$$

If the eigenvalue a is degenerate and P_a is the projector onto its subspace,

$$P(a) = \langle\psi|P_a|\psi\rangle.$$

For a continuous position measurement on the line, in the position representation $\psi(x) = \langle x \,|\, \psi\rangle$,

$$P(x \in [a,b]) = \int_a^b |\psi(x)|^2 \,\mathrm{d}x.$$

The integral is a probability. It is dimensionless. $|\psi|^2$ is a density, per length.

Born's 1926 proposal was that the wave function gives probabilities of outcomes, not a classical charge density. In the modern operator form the rule is an extra sentence. It is not a line of the Schrödinger equation.

5.2 Each symbol in the spin example

Take $|\psi\rangle = |{+}\text{x}\rangle$ and the observable σ_z.

Symbol	Value

$\lvert a_+\rangle$	$\lvert\mathrm{up}\rangle = \binom{1}{0}$. The outcome label $+1$.
$\lvert a_-\rangle$	$\lvert\mathrm{down}\rangle = \binom{0}{1}$. The outcome label -1.
$\langle \mathrm{up} \mid +x\rangle$	$\frac{1}{\sqrt{2}}$.
$P(+1)$	$\left\lvert\frac{1}{\sqrt{2}}\right\rvert^2 = \frac{1}{2}$.
$P(-1)$	$\frac{1}{2}$.
$\langle\sigma_z\rangle$	$$(+1)\left(\frac{1}{2}\right) + (-1)\left(\frac{1}{2}\right) = 0$$. The average over many runs. Not a run.

Use: compute the inner product of the state with each outcome vector, square the modulus, and treat that number as the probability of printing that eigenvalue. Over N independent runs the count of $+$ is expected near $\frac{N}{2}$, with fluctuations of order $\frac{\sqrt{N}}{2}$. One run prints $+1$ or -1, never $\frac{1}{2}$.

6 The measurement postulate

6.1 The update

Von Neumann's projection postulate, in the form used for a nondegenerate outcome a_n that has occurred, replaces the state by the corresponding eigenvector:

$$\lvert\psi\rangle \to \lvert a_n\rangle.$$

Lüders' form covers degeneracy and incomplete measurements. If P_n projects onto the subspace associated with the result that occurred,

$$\lvert\psi\rangle \to \frac{P_n\lvert\psi\rangle}{\sqrt{\langle\psi\rvert P_n\lvert\psi\rangle}}.$$

The denominator restores the norm. It depends on $\lvert\psi\rangle$. The map is therefore nonlinear. It is also stochastic: which P_n is applied is not fixed by H.

For the example, if σ_z returns $+1$,

$$\lvert{+\mathrm{x}}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \to \lvert\mathrm{up}\rangle = \begin{pmatrix}1\\0\end{pmatrix}.$$

The relative phase is gone. The component along $\lvert\mathrm{down}\rangle$ is gone. Nothing in $U(t) = \exp\bigl(-iH\frac{t}{\hbar}\bigr)$ performs that deletion. For the Hamiltonian of Example A, $U(t)\lvert{+\mathrm{x}}\rangle$ has both components of modulus $\frac{1}{\sqrt{2}}$ at every t.

6.2 Why the update is required by what is observed

Consider an ideal Stern–Gerlach measurement, or any apparatus that is supposed to be repeatable for the quantity it measures.

1. Prepare many spins in $\lvert{+\mathrm{x}}\rangle$.
2. Measure σ_z. About half the runs read $+1$, half read -1. That matches Born's rule. Nobody, on any run, finds a detector that has clicked "half."
3. Take only the runs that read $+1$, and measure σ_z again, immediately, with no new field that would rotate the spin.
4. The second reading is $+1$ on those runs, not a fresh fair coin.

If the state is left at $\lvert{+\mathrm{x}}\rangle$, or at $U(t)\lvert{+\mathrm{x}}\rangle$ for the Hamiltonian above, Born's rule applied a second time still gives $P(+1) = \frac{1}{2}$. That is not what a repeatable measurement does. The algorithm therefore inserts the

update before the second application of Born's rule. After the update, $P(+1) = |\langle \text{up} \,|\, \text{up} \rangle|^2 = 1$. The second click agrees with the first.

The same point in a path language: a photon in a balanced interferometer is not observed to arrive as a half-click in each port. It arrives in one port. If both paths carry amplitude and no path mark is made, the port probabilities need not be $\frac{1}{2}$; they can be 1 and 0 by interference. If a path mark is made and the mark says "upper," the interference is gone and each port is then $\frac{1}{2}$. The mark is definite. The $\frac{1}{2}$ is the weight of that mark in the ensemble, or the probability assigned before the mark.

6.3 Why the update is not the Schrödinger equation

Put the two maps side by side on the same vector $|{+\text{x}}\rangle$.

	Schrödinger, $H = (\hbar\frac{\omega}{2})\sigma_z$	**Update on outcome $+1$**
Input	$\|{+\text{x}}\rangle$	$\|{+\text{x}}\rangle$
Output	$\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega\frac{t}{2}} \\ e^{i\omega\frac{t}{2}} \end{pmatrix}$	$\|\text{up}\rangle$
Linear?	Yes. $U(\alpha\|u\rangle + \beta\|v\rangle) = \alpha U\|u\rangle + \beta U\|v\rangle$.	No. The factor $\frac{1}{\sqrt{P}}$ depends on the state, and the branch depends on the outcome.
$P(+1)$ after	$\frac{1}{2}$	1
Both components kept?	Yes.	No.

There is no time t and no ω at which the left-hand output equals $|\text{up}\rangle$. The two maps disagree on the only fact the second measurement checks.

Including the apparatus does not remove the split. Let $|\text{R}\rangle$ be a ready pointer. A linear interaction of the von Neumann type can be arranged so that

$$|{+\text{x}}\rangle|\text{R}\rangle \to \frac{1}{\sqrt{2}}(|\text{up}\rangle|\text{R+}\rangle + |\text{down}\rangle|\text{R-}\rangle),$$

and this joint vector is exactly what the Schrödinger equation of the enlarged system produces if the coupling is written into H. The joint vector still contains both pointer readings. A laboratory record contains one. To match the record, the algorithm again replaces the sum by one term and renormalizes. The chain can be lengthened by a room, a notebook, or a second observer. Linearity keeps every term. The update deletes all but one. That is the measurement problem in its narrow, mathematical form.

6.4 Maudlin's three sentences

Maudlin (1995) isolates the contradiction as three sentences that cannot all be true:

1. The wave function of a system is complete. Nothing further is needed to say what is true of that individual.
2. The wave function always evolves by a linear equation of Schrödinger's type.
3. Measurements have definite outcomes. One reading occurs, not a sum of readings.

Sentence 2 applied to the joint system yields the entangled sum of pointer readings. Sentence 3 says one reading occurred. Sentence 1 says there is nothing else in the world that could pick which reading is the real one. Drop 1 and you are in Einstein's demand, or in a hidden-variable theory. Drop 2 and you add a collapse law, a stochastic or nonlinear modification of the equation. Drop 3 and you keep every term, as in Everett's reading, and you owe an account of why a record looks single. The orthodox algorithm refuses to drop any sentence in words, and in practice drops 2 at the moment a result is written down.

7 Einstein's stance

The 1935 argument of Einstein, Podolsky, and Rosen is not a claim that quantum theory contradicts itself. It is a completeness argument. If one grants a locality assumption — a measurement on one system does not instantly change the real state of a distant system — and if one grants that the algorithm's certain predictions are elements of reality, then the wave function of the distant system is not the whole reality. Einstein later separated himself from the particular "element of reality" wording. The 1949 Reply to Criticisms keeps the demand and changes the illustration.

The illustration is the radioactive atom and the mark it writes. The ψ-function gives a probability that the mark is written in a given interval. It does not say, of this atom, when the mark is written. Einstein's conclusion, as 2026a uses it, is a successor-theory demand: a complete description of the individual situation, whose laws, per se, have nothing to do with statistics. Statistical quantum theory would then stand to that description as statistical mechanics stands to classical mechanics.

A letter of 31 October 1949, now published in the Cooper correspondence, makes the fork explicit. Einstein did not think quantum theory internally inconsistent. He thought that if ψ is complete, then the real state of a distant subsystem depends on the kind of measurement made on the near one, which is action at a distance; and that if there is no such action, ψ is not complete. The Reply and the letter agree on the fork. They do not yet know Bell's theorem. They do not say that the click statistics are an illusion.

2026a takes the single-system half of this fork and writes $T^*(S)$. 2026b refuses to replace the fork by unification-as-quantization. Neither paper pretends that a shared unread remainder, evolving by a local law, has been shown to reproduce distant correlations stronger than Bell's bound. This paper is the place that limitation is calculated.

8 Bell, Aspect, and what was actually closed

8.1 The inequality, without a wave function

Two sides, A and B. Each trial has settings a or a' on the left and b or b' on the right, and outcomes $A, B \in \{+1, -1\}$. Write $E(a, b)$ for the average of the product AB on the trials with that pair of settings. Define

$$S = E(a, b) + E(a, b') + E(a', b) - E(a', b').$$

Assume: there is a variable λ, shared or not, such that the outcome on the left is a function of λ and the left setting only, the outcome on the right is a function of λ and the right setting only, and the choice of settings is independent of λ. Then each trial contributes a value whose absolute value is at most 2, because

$$A(B + B') + A'(B - B')$$

has one of the parentheses equal to 0 and the other equal to ± 2 whenever $B, B' = \pm 1$. Averaging preserves the bound:

$$|S| \leq 2.$$

This is the CHSH inequality (Clauser, Horne, Shimony, Holt 1969). No inner product was used. The bound is a constraint on counts.

8.2 What the singlet predicts

For the singlet, quantum theory gives $E(\theta) = -\cos\theta$ if θ is the angle between the two analyzer directions. The choice $\theta_{ab} = \theta_{ab'} = \theta_{a'b} = \frac{\pi}{4}$ and $\theta_{a'b'} = 3\frac{\pi}{4}$ yields four correlations of magnitude $\frac{1}{\sqrt{2}}$ and

$$S = 2\sqrt{2} \approx 2.828.$$

Tsirelson's bound says $2\sqrt{2}$ is the largest modulus quantum theory can assign to this combination. The number sits strictly above 2. A local model of the kind just defined cannot produce it, not as a matter of technology, but as a matter of the inequality.

A shared unread remainder in the sense of 2026a is a λ. If each click is fixed by that remainder and the local setting, the same bound applies. Correlated phases of two lumps explain correlated exit times. They do not explain $S = 2\sqrt{2}$.

8.3 The experiments, as constraints on counts

Early tests, including Aspect, Grangier, and Roger (1982), violated the inequality and left exits. Inefficient detectors: a local model could refuse to count the runs that would have restored $|S| \leq 2$ (the detection loophole). Settings chosen too close in time or space to the source: a local influence could in principle have correlated λ with the settings (the locality, or freedom-of-choice, loophole).

The exits that matter were closed in 2015.

- Hensen et al., *Nature* (2015): two electron spins, 1.3 km apart, entangled by swapping. They never interacted with each other. Every spin was read, so missed detections are not the excuse. Over 245 trials, $S = 2.42 \pm 0.20$, and the probability of a local-realist model producing so large a violation was at most 0.039, memory in the devices allowed for. The sample is small. A later comment (Adenier and Khrennikov) argued that this particular table also strains no-signaling. That is a caveat about those 245 trials. It is not a property of the photon experiments next.
- Giustina et al., *Phys. Rev. Lett.* 115, 250401 (2015): photons, locality and detection closed together. The probability of the observed violation under local realism at most 3.74×10^{-31}, about 11.5 standard deviations.
- Shalm et al., *Phys. Rev. Lett.* 115, 250402 (2015): the same class of closure, independently.
- Rosenfeld et al., *Phys. Rev. Lett.* 119, 010402 (2017): atoms, both loopholes closed, with much tighter statistics than the 245-trial run.

Handsteiner et al. (2017) set the analyzers from Milky Way starlight and still violated the inequality, fair sampling assumed, locality closed. The quasar runs (Rauch et al. 2018) push any local conspiracy that coordinated settings with pairs back billions of years, under the assumption that the astronomical color was fixed at emission. They do not logically forbid a correlation laid down in the initial conditions of the universe. They do forbid a conspiracy arranged in the laboratory that afternoon.

No-signaling is the companion fact. In the high-statistics tests the marginal counts on one side do not depend, within error, on the distant setting. The tables are more correlated than $|S| \leq 2$ allows, and they are not a telephone.

Greenberger, Horne, and Zeilinger (1989) replace the inequality, for ideal correlations of three or four particles, by a contradiction in a single combination of outcomes: pre-existing local values cannot match all the certain predictions at once. Pan et al., *Nature* (2000), is the corresponding three-photon experiment. Real visibilities are not perfect. The logical point survives: the conflict is not only an average.

Entanglement swapping, which the Delft test uses, blocks a lazy reading of “they shared a remainder at birth.” The two spins whose clicks violate the inequality never occupied one preparation together. Whatever is correlated was correlated through the partner photons and the heralding event.

8.4 What these tests are not

They are not a photograph of a vector. They are not a detection of collapse as a process. They are not a message sent outside the light cone. State tomography, concurrence, and “fidelity to the singlet” are reconstructions inside the algorithm: the same Born rule that was used to predict the counts is used to name a vector that fits the counts. That naming is circular if it is offered as independent evidence that the vector was the object on the bench. The inequality violation is not circular in that way. It is a statement about frequencies and settings.

9 Prediction, explanation, and the suspicion of self-deception

Physics of this kind is unusually good at prediction and unusually thin as an account of one run. The thinness is not a scandal by itself. Statistical mechanics is thin about one molecule and is none the worse, because a

complete mechanical description is available underneath. Einstein's demand is that quantum theory be thin in that same way, not thin because the individual has no further description.

The suspicion that we fool ourselves has a precise form. It is not "the detectors were misread." It is: we speak as if a linear superposition were observed, because the bookkeeping that predicts the frequencies is a linear vector, and we then treat the update — the step that actually matches the single record — as a minor extra. The weights are doing predictive work. The story about the unmeasured individual is not.

9.1 Where the suspicion is right

1. **Two laws, one advertisement.** Textbooks say the Schrödinger equation is the dynamics. The calculation that matches a repeatable measurement uses the equation and then a second map that the equation cannot produce. If the second map is omitted, the prediction for the immediate re-measurement is wrong. If it is included, linearity has been suspended for that step. Both facts can be written without any philosophy. They are the tables in the measurement section.
2. **A half-click is never the observation.** $P = \frac{1}{2}$ is not a property one finds on a single detector. It is a weight. Saying that the spin was "half up and half down" uses the weight as if it were the record. The record is one eigenvalue. The orthodox algorithm itself says so, in the update, and then the surrounding prose often takes it back.
3. **The reconstructed state is not a sighting.** Tomography fits $|\psi\rangle$ to frequencies. It does not display $|\psi\rangle$ before the clicks. Reporting "we observed the entangled state" reports a fit. The fit can be excellent and still not be an object that was seen.
4. **Interference is real and is not a photograph either.** Blocking one path in an interferometer changes the port frequencies. Any completion has to carry relative phase, or it will get those frequencies wrong. That is a constraint on $S_\perp$, not a sighting of ψ. The Rank-1 analog already carries phase, locally, in θ_k. That does not address distant S.

9.2 Where the suspicion is wrong

1. **The Bell tables are not an artifact of writing a linear ψ.** The bound $|S| \leq 2$ can be stated, and was stated above, with no wave function. The counts in Giustina et al. and Shalm et al. violate it by many standard deviations with the detection and locality loopholes closed. Throwing away ψ as ontology does not throw away the counts.
2. **A local shared remainder does not become adequate once ψ is called epistemic.** If the remainder and the local setting fix each outcome, CHSH applies. Calling λ "unread field" instead of "hidden variable" does not change the algebra.
3. **There is no new gap, in the tables reviewed here, that restores local realism with free settings.** The residuals are these: freedom-of-choice can be pushed back to cosmological initial conditions and not abolished; the 245-trial Delft table is statistically modest and has been disputed on no-signaling; fair sampling remains an assumption in the cosmic runs, which did not close detection. None of these residuals is a reconstruction of $|S| \leq 2$ for the 2015 photon experiments.

Pusey, Barrett, and Rudolph (2012) constrain a further hope: that ψ is only knowledge about a deeper state, with independently prepared systems. Under their assumptions the deeper state has to determine ψ. A program that correlates remainders at a common preparation denies preparation independence on purpose, so that theorem does not by itself forbid 2026a. Bell's factorization still applies to distant outcomes. The two theorems close different doors.

9.3 What would count as fooling ourselves about entanglement

Only this: to report a Bell violation as if it were a picture of a pre-measurement superposition, or to report a local completion as if correlated phases had already matched $2\sqrt{2}$. The first over-reads the counts. The second under-reads the inequality. This paper declines both.

The predictive success of the algorithm is not in dispute. Born's rule plus the update plus unitary evolution between updates is how the frequencies are obtained, and the frequencies come out right, including the Bell

frequencies. The explanatory gap is that this success is achieved by a law that cannot be the only law, and by a vector that is never the record. That is a reason to ask for a completion. It is not a reason to discard the tables the completion must eventually meet.

10 What the program may say, after this exhibition

Taken at face value, 2026a already says the following about one system. If S is given, T^* is a number. If only S_{coarse} is given, the times form the pushforward of a measure on $S_\perp$. Probability is not a term in the law. A second look that reveals more of the true remainder must narrow the time. None of that mentions a bra, a ket, or a projector.

Taken at face value, the same paper does not say the following. It does not assign outcomes to a pair of distant analyzers. It does not claim $|S| \le 2$ for those analyzers, and it does not claim $S = 2\sqrt{2}$. Step 6 remains the open question: whether two distant lumps can inherit correlated remainders from a common preparation without a nonlocal term in the field equation.

This exhibition adds a boundary condition on that step.

1. A mature completion may refuse ontological ψ, refuse collapse as a second process, and treat Born weights as pushforwards of unread remainder. Nothing in the measurement problem forbids that refusal. The repeatability of a single apparatus becomes a fact about the field after the mark, not an extra map.
2. The same completion may not stop at “the phases were correlated.” If each click is a field fact fixed by the local field and the local setting, and if settings are independent of the remainder, the CHSH derivation applies and $2\sqrt{2}$ is unreachable.
3. Entanglement swapping means the two records that violate the bound need not come from two lumps that shared a preparation. A remainder assigned only at the birth of those two lumps is the wrong ontology for that experiment.
4. No-signaling is part of the empirical constraint. Marginals on one side do not carry the distant setting. A nonlocal term thrown in to raise S has to keep that fact, or it has failed a different test.
5. The forks that remain are the ones Bell left open: a nonlocal structure in the field; a correlation between settings and remainder old enough that the cosmic tests have not excluded it; or a retrocausal reading in which the later setting is already relevant to what is true of the field. 2026b declines cheap nonlocal extras as Einstein’s architecture. This paper does not smuggle one back in. It records that the local, free-setting branch does not graduate.

No new experimental gap was found that would let the local branch graduate unnoticed. The gap that fits “are we fooling ourselves?” is the one inside the algorithm: the update is required by the records, the Schrödinger equation does not supply it, and the prose of superposition describes a vector the records never are.

11 Worked sheet

The sheet is the spin calculation with every number, so that a reader can repeat it without the prose.

Given. $|\psi\rangle = |{+\mathrm{x}}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, observable σ_z, Hamiltonian $H = (\hbar\frac{\omega}{2})\sigma_z$, ω any real number.

Born, first measurement.

$$P(+1) = |\langle \mathrm{up} \,|\, \psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}.$$

$$P(-1) = \frac{1}{2}.$$

Schrödinger at any t.

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega\frac{t}{2}} \\ e^{i\omega\frac{t}{2}} \end{pmatrix}.$$

$$P_t(+1) = \frac{1}{2}.$$

The equation has not selected an outcome.

Update if the outcome is $+1$.

$$|\psi\rangle \to |\mathrm{up}\rangle.$$

$$P_{\mathrm{second}}(+1) = 1.$$

$$P_{\mathrm{second}}(-1) = 0.$$

Refusal of the update. Apply Born's rule to $|\psi(t)\rangle$ again:

$$P_{\mathrm{second}}(+1) = \frac{1}{2}.$$

This disagrees with repeatability. Therefore the algorithm that matches the second click is not the Schrödinger solution.

Singlet correlation, one angle.

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\mathrm{up\ down}\rangle - |\mathrm{down\ up}\rangle).$$

Measure σ_z on both sides. The two terms give products (-1) and (-1), so $E = -1$ when the axes coincide, in agreement with $-\cos 0$. The four-angle combination quoted above is $2\sqrt{2}$, which is greater than 2.

Local bound, one line. For outcomes ± 1, $|AB + AB' + A'B - A'B'| = |A(B + B') + A'(B - B')| \leq 2$ on every trial. The average cannot exceed 2. Quantum theory's $2\sqrt{2}$ is outside that set. Shared λ does not cross it.

12 Sources

The list is the spine used above, not a bibliography of the measurement problem. Where a page number is not needed for the claim, none is invented.

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